Closing Wed: HW_6A, 7A (7.5/7.7, 7.8)
Exam 2 is Thursday: 6.3-6.5, 7.1-7.5, 7.7, 7.8

A Very Brief Exam 2 Review

Integration!

1. The 4 special methods

By parts: $xe^{3x}, x\sin(4x), x^2\cos(5x), \frac{\ln(x-1)}{x^2}, x\tan^{-1}(x).$

Trig: $\sin^3(x)\cos(x), \cos^4(x), \sec^3(2x)\tan^3(2x)$

Trig sub: $\frac{1}{x\sqrt{x^2-9}}, \frac{1}{(4-x^2)^{3/2}}, \sqrt{x^2+6x+10}.$

Part Frac: $\frac{x+2}{(x-1)(x-3)}, \frac{4x}{(x-1)^2(x-3)}, \frac{5}{x(x^2+4)}, \frac{x^2}{x+7}, \frac{x-3}{x^2+8x+20}$

2. Substitution and Simplifying

Try $u = \sqrt{x}$, u = inside, u = e^x, u = trig.
Know essential toolbox of trig facts, such as $\tan(x) = \frac{\sin(x)}{\cos(x)}$, $\sec(x) = \frac{1}{\cos(x)}$.
square identities, half-angle.

3. Improper Integrals:

a) Rewrite as a limit!!
b) Integrate
c) Take limit

4. Trapezoid/Simpson Rules

a) Set up integral, then compute width and label tickmarks.
b) Use formula.

Random Integrals from Old Finals:

1. $\int \frac{1-x}{\sqrt{1-x^2}}\,dx$
2. $\int \frac{x^2-x+8}{x^3+4x}\,dx$
3. $\int 2x\ln(x+5)\,dx$
4. $\int \cos^3(x)\,dx$
5. $\int_0^2 \frac{1}{\sqrt{x^2+2x+4}}\,dx$
6. $\int_1^3 \frac{1}{x^2+x^3}\,dx$
7. $\int \tan^2 x \sec^4(x)\,dx$
8. $\int \frac{1}{\left(1+\sqrt{x}\right)^3}\,dx$
9. $\int \sin(x)\sqrt{\cos(x)}dx$

Random Improper Integrals:

1. $\int_1^2 \frac{x}{\sqrt{x-1}}\,dx$
2. $\int_{-3}^{\infty} xe^{-x}\,dx$
3. $\int_1^{\infty} \frac{1}{\sqrt{x}(1+x)}\,dx$

Approximation:

1. Use Simpson's Method with n = 4 subdivision to approximate the value of

$$\int_0^4 \sqrt{1+4x^4}\,dx$$

5. New Applications

a) **Volumes of Revolution**

V = $\int_a^b 2\pi(Radius)(Height)(Thickness)$

V = $\int_a^b (\pi(Outer\ R)^2 - \pi(Outer\ R)^2)(Thick)$

b) **Average value** = $\frac{1}{b-a}\int_a^b f(x)dx$

c) **Work** = $\int_a^b (Force)(Dist)$

(i) If f(x) = "force formula at x",
(Spring, leaky bucket, …)
then
Force = f(x), *Dist* = Δx:

$$\text{Work} = \int_a^b f(x)dx$$

(ii) *Chain/Cable*: k = force/length
If you label top: x = 0, then
Force = k Δx, *Dist* = x,

$$\text{Work} = \int_a^b k\ \text{x}\ dx$$

(iii) *Pumping*: k = force/volume
If bottom is y = 0 and top is y = b,
Force = k(Area)Δy, *Dist* = b – y

$$\text{Work} = \int_a^b k(\text{Area})(b-y)dy$$

Applications from old tests:

1. Consider the region bounded by $y = \sqrt{x}$, the horizontal line y = 2, and the y-axis.
(a) Find the volume obtained by rotating about the x-axis.
(b) Find the volume obtained by rotating about the horizontal line y = 5.

2. Find the average value of $\cos^3(x)$ on the interval 0 to $\pi/2$.

3. A spring has natural length of 30 cm from the wall. It requires 2 J of work to stretch it from 40 cm to 45cm (from the wall). How far beyond its natural length will a force of 64 N keep the spring stretched?

4. A 1600 lb elevator is suspended by a 200 ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?

5. A rope is used to pull a bucket full of water up from a well that is 10 m deep. The rope has a total mass of 5 kg. The bucket has a mass of 11 kg. Find the total work done in lifting the bucket to the top (Recall: Accel. due to gravity is 9.8 m/s^2)

6. A well is in the shape of a cylinder of radius 1 meter and depth 8 meters. It is half full of water. Find the word to pump all the water to the top.
(Recall: Water weighs 9800 N/m^3)

7. The portion of the graph $y = x^2 / 9$ between $x = 0$ and $x = 3$ is rotated about the y-axis to form a container. The container is full of a liquid that has density 100 lbs/ft^3. Find the work required to pump all the liquid to the top of the container.